Swan Christian College

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 1&2

Section One: Calculator-free

SOLUTIONS

WA student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

The weekly time sheet for a part time worker is shown below.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Hours worked	-	-	2.5	4.5	5	4	-

The worker is paid $\$20$ per hour, with time and a half paid for weekend shifts.

(a) Determine the gross weekly pay for this worker. (3 marks)

Solution
Total hours paid: $h = 2.5 + 4.5 + 5 + 4 \times 1.5$ $= 12 + 6$ $= 18$ Pay: $P = 18 \times 20$ $= \$360$
Specific behaviours
✓ correctly applies time and a half ✓ total hours worked ✓ correct pay

(b) The following week the gross weekly pay for the worker came to $\$270$ but as they met a performance target, they were awarded a 15% bonus. Determine the amount of their bonus. (2 marks)

Solution
$270 \times 10\% = 27.00$ $270 \times 5\% = 13.50$ $\text{Bonus} = 27.00 + 13.50$ $= \$40.50$
Specific behaviours
✓ indicates appropriate method ✓ correct amount

See next page

Question 2 **(5 marks)**

The table below displays a selection of variables and sample responses from a study dataset.

Visits to clinic	Gender	Height	Blood group	BMI	Feeling happy?	Number of siblings	Income level
2	F	1.66	AB	19.2	Disagree	2	Low
4	M	1.82	O	18.4	Strongly agree	0	High
5	M	1.74	B	32.7	Strongly disagree	1	V High
1	F	1.80	AB	24.1	Agree	2	Medium

(a) Give the name of a variable from the table that is classified as

(i) categorical and nominal. (1 mark)

Solution
Gender OR Blood group
Specific behaviours
✓ correct example

(ii) numerical and discrete. (1 mark)

Solution
Visits to clinic OR Number of siblings
Specific behaviours
✓ correct example

(iii) categorical and ordinal. (1 mark)

Solution
Feeling happy? OR Income level
Specific behaviours
✓ correct example

(b) The BMI variable is to be replaced with a Body Type variable according to the following table:

BMI	< 18.5	$18.5 - 24.9$	$25 - 29.9$	≥ 30
Body Type	Underweight	Normal	Overweight	Obese

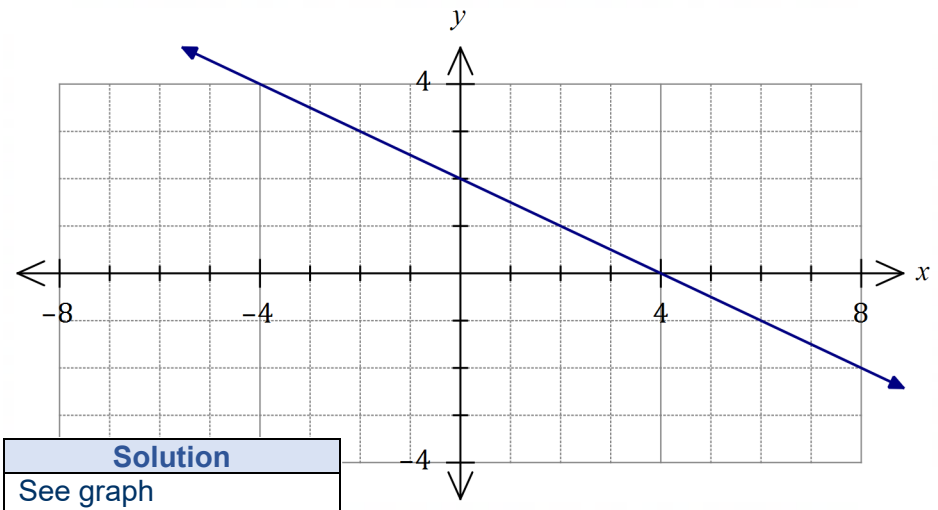
Explain whether Body Type will have the same variable classification as BMI. (2 marks)

Solution
No - BMI is a numerical and continuous variable but Body Type is a categorical and ordinal variable.
Specific behaviours
✓ states no with reasons ✓ indicates that numerical and continuous becomes categorical and ordinal

See next page

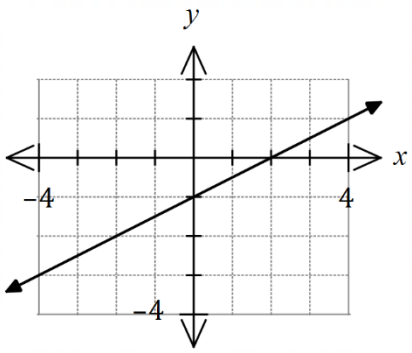
Question 3 **(7 marks)**

(a) Construct the graph of $x + 2y = 4$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ x-intercept
✓ y-intercept
✓ straight-line

(b) The graph of $y = a + bx$ is shown at right.



(i) Determine the value of the constant a and the value of the constant b. (2 marks)

Solution
$a = -1, \quad b = \frac{1}{2} = 0.5$
Specific behaviours
✓ correct value of a
✓ correct value of b

(ii) State with justification whether the graph of $y = a + bx$ passes through the point with coordinates $(30, 16)$. (2 marks)

Solution
$y = -1 + \frac{1}{2}(30) = 14$ No, does not pass through the point as when $x = 30$, y should be 14, not 16, to lie on the line.
Specific behaviours
✓ states no
✓ justifies using coordinates

See next page

Question 4 **(8 marks)**

(a) A mixture of linear and non-linear equations are shown below. Using a pencil, draw a circle around each linear equation. (2 marks)

$2x = 9,$ $x^2 = 9,$ $2(x+1) = 3^x,$ $3x = 2(x+1),$ $x - 2 = 5x.$

Solution
Circles around equations $1, 4, 5$.
Specific behaviours
✓✓ - subtract one ✓ each error or omission

(b) Solve the equation $2(2x+3) - 5 = 3 - 3(3-x)$ for x. (2 marks)

Solution
$4x + 6 - 5 = 3 - 9 + 3x$ $x = -7$
Specific behaviours
✓ correctly expands both sides ✓ correct solution

(c) Ana, Bo and Cole are reading the same book. Ana has read x pages, Bo has read 30 more pages than Ana and Cole has read twice as many pages as Bo. The mean number of pages read by the three friends is 42.

(i) Use the above information to write an equation in terms of x. (2 marks)

Solution
Bo has read $x + 30$ pages and Cole has read $2(x+30)$ pages. Hence: $\frac{x + x + 30 + 2(x+30)}{3} = 42$
Specific behaviours
✓ expressions for Bo and Cole in terms of x ✓ equation using mean

(ii) Determine how many pages of the book Ana has read. (2 marks)

Solution
$2x + 30 + 2x + 60 = 3 \times 42 = 126$ $4x = 36$ $x = 9$ Ana has read 9 pages.
Specific behaviours
✓ expands and simplifies equation ✓ states correct number of pages read by Ana

See next page

Question 5 **(7 marks)**

(a) 120 and 86 purchases were made at a shop in the morning and afternoon respectively. In the morning, half of the purchases were made with EFTPOS, 23 were cash and the remainder using a pay later scheme. In the afternoon, 16 were made with cash and the remainder evenly split between EFTPOS and pay later.

Represent the figures for the time of day and payment type in a labelled 2×3 matrix. (2 marks)

Solution
$\begin{matrix} & E & C & PL \\ M & \begin{bmatrix} 60 & 23 & 37 \end{bmatrix} \\ A & \begin{bmatrix} 35 & 16 & 35 \end{bmatrix} \end{matrix}$
Specific behaviours
✓ any labelled 2×3 matrix
✓ correct matrix

(b) Simplify

(i) $12I - 3\begin{bmatrix} 4 & 1 \\ -5 & 0 \end{bmatrix}$, where I is the 2×2 identity matrix. (3 marks)

Solution
$\begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 12 & 3 \\ -15 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 15 & 12 \end{bmatrix}$
Specific behaviours
✓ shows multiple of identity matrix
✓ correct scalar multiplication
✓ correct matrix

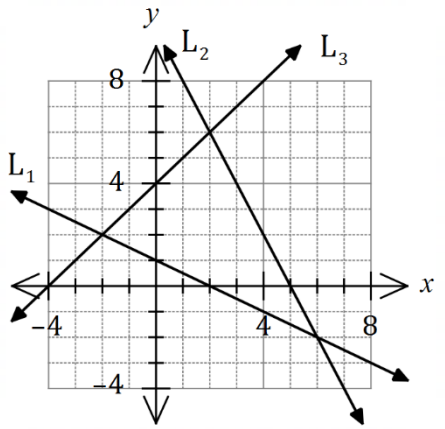
(ii) $\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix}$. (2 marks)

Solution
$\begin{bmatrix} 7 & -10 \\ 14 & -2 \end{bmatrix}$
Specific behaviours
✓ one correct row or column
✓ correct matrix

See next page

Question 6 **(7 marks)**

(a) Three straight-lines L_1, L_2 and L_3 are shown on the graph at right.



Explain how the graph can be used to solve the simultaneous equations

$y = -2x + 10$ and $y = x + 4$

and state their solution. (3 marks)

Solution
Read from the graph the point of intersection of L_2 and L_3. The solution is $x = 2, y = 6$.
Specific behaviours
✓ states point of intersection of two lines ✓ chooses correct lines ✓ correct solution

(b) A cafe can buy disposable coffee cups from two suppliers, P and Q. Supplier P charges a delivery fee of $\$8$ plus 3 cents per cup whilst supplier Q has no delivery fee but charges 5 cents per cup. Let y be the total cost in cents of buying x cups from a supplier.

(i) Write an equation relating x and y for supplier P. (1 mark)

Solution
$y = 800 + 3x$
Specific behaviours
✓ correct equation

(ii) Write an equation relating x and y for supplier Q. (1 mark)

Solution
$y = 5x$
Specific behaviours
✓ correct equation

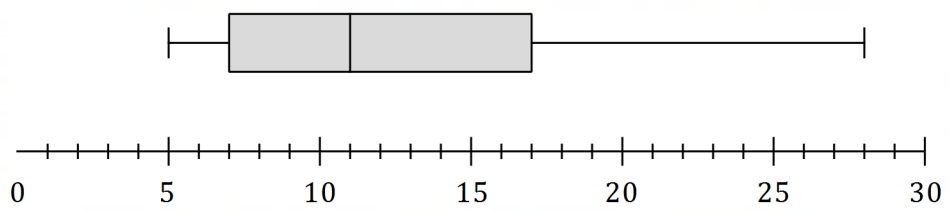
(iii) Determine the number of cups for which the total cost is the same for both suppliers and state what this total cost is. (2 marks)

Solution
$800 + 3x = 5x$ $2x = 800$ $x = 400$ c $y = 5 \times 400 = \$20$ Total cost is the same when 400 cups are bought. The cost of 400 cups is $\$20$.
Specific behaviours
✓ solves equations for x ✓ states number of cups and total cost

See next page

Question 7 **(7 marks)**

The box plot below represents the distribution of the number of flights per day that were subject to a delayed departure at a large airport.



(a) Determine, for this distribution, the

(i) median. (1 mark)

Solution
$\text{median} = 11$
Specific behaviours
✓ correct value

(ii) range. (1 mark)

Solution
$\text{range} = 28 - 5 = 23$
Specific behaviours
✓ correct value

(iii) interquartile range. (1 mark)

Solution
$\text{IQR} = 17 - 7 = 10$
Specific behaviours
✓ correct value

(b) Describe a feature of the box plot that indicates the mean of the distribution will be greater than the median. (1 mark)

Solution
The width of right hand side of box plot is much greater than the width of the left hand side.
Specific behaviours
✓ describes positive skew

(c) Construct a possible ordered list of seven whole numbers that would result in the box plot shown. (3 marks)

Solution
$5, 7, [7-11], 11, [11-17], 17, 28$ $[a-b]$ *indicates any number in range, inclusive*
Specific behaviours
✓ minimum, maximum, median ✓ second and second last numbers (LQ and UQ) ✓ correct set of 7 ordered whole numbers

See next page

Question 8 **(6 marks)**

(a) A formula used to estimate the speed, S km per hour, of a car that skids to a halt is shown below. It uses the length of the skid d m and the friction coefficient f that varies with the type of road and weather.

$$S = 3.5\sqrt{10fd}$$

	f	
Road Type	Wet	Dry
Bitumen	0.50	0.90
Concrete	0.45	0.85
Unsealed	0.35	0.60

Estimate the speed of a car that skidded to a halt over a distance of 13.5 m on a dry unsealed road. (3 marks)

Solution
$S = 3.5\sqrt{10 \times 0.60 \times 13.5}$ $= 3.5\sqrt{81}$ $= 3.5 \times 9$ $= 31.5$ km/h
Specific behaviours
✓ uses correct value of f ✓ simplifies radicand correctly ✓ correct speed

(b) In baseball statistics, slugging average SLG is a measure of the batting productivity of a hitter. It is calculated from the number of singles S, doubles D, triples T, home runs H and at bats AB using the formula

$$SLG = \frac{S + 2D + 3T + 4H}{AB}$$

A hitter who had been at bat 40 times had a slugging average of 0.4. Given that they had hit one home run, one triple, and five singles, determine their number of doubles. (3 marks)

Solution
$0.4 = \dfrac{5 + 2x + 3(1) + 4(1)}{40}$ $0.4 \times 40 = 2x + 12$ $2x = 16 - 12$ $x = 2$ They hit 2 doubles.
Specific behaviours
✓ substitutes correctly ✓ shows at least two correct steps to simplify ✓ correct number

End of questions

Swan Christian College

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 1&2

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

The number of shares that three investors A, B and C held in the three companies X, Y and Z is shown in the following matrix M.

$$M = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{bmatrix} 510 & 510 & 360 \\ 240 & 390 & 530 \\ 310 & 220 & 490 \end{bmatrix} \end{array}$$

The current market value of one share in X, Y and Z is $\$1.15, \$2.35$ and $\$1.90$ respectively.

(a) Represent the market values in matrix V and using your calculator, or otherwise, calculate the matrix product MV and describe what the entries in the product represent. (3 marks)

Solution
$V = \begin{bmatrix} 1.15 \\ 2.35 \\ 1.90 \end{bmatrix}$ $MV = \begin{bmatrix} 2\,469.00 \\ 2\,199.50 \\ 1\,804.50 \end{bmatrix}$ Entries are the total value of each investor's shares.
Specific behaviours
✓ matrix V ✓ correct matrix ✓ correct description

The dividends per share paid by companies X, Y and Z are $9\text{c}, 23\text{c}$ and 5c respectively.

(b) Use a matrix method to determine the dividend that each investor will receive on their shares. (3 marks)

Solution
$D = \begin{bmatrix} 0.09 \\ 0.23 \\ 0.05 \end{bmatrix}$ $MD = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{bmatrix} 181.20 \\ 137.80 \\ 103.00 \end{bmatrix}$
Specific behaviours
✓ indicates dividend matrix D ✓ indicates product MD ✓ correct product

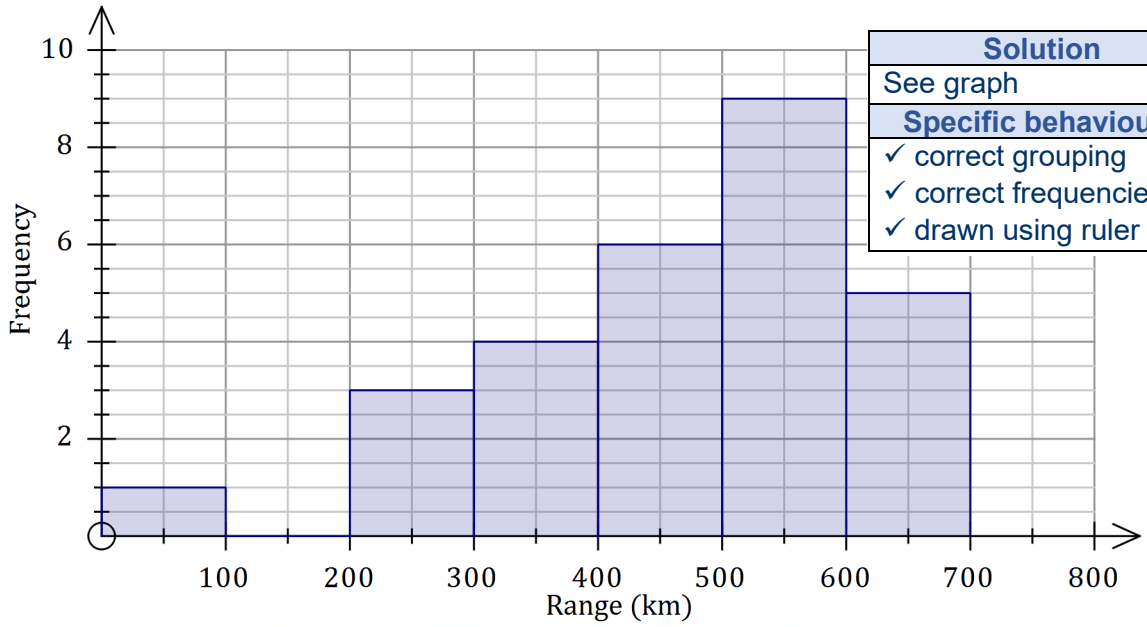
See next page

Question 10 **(7 marks)**

The results of a recent investigation into the range of consumer electric vehicles are summarised in the following table.

Range of EV (km)	Frequency
$0 \leq x < 100$	1
$100 \leq x < 200$	0
$200 \leq x < 300$	3
$300 \leq x < 400$	4
$400 \leq x < 500$	6
$500 \leq x < 600$	9
$600 \leq x < 700$	5

(a) Construct a histogram of this data on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ correct grouping
✓ correct frequencies
✓ drawn using ruler

(b) Calculate the mean and standard deviation of the range. (2 marks)

Solution
$\bar{x} = 468$ km, $\sigma_x = 147$ km
Specific behaviours
✓ correct mean
✓ correct standard deviation

(c) Describe the shape and modality of the distribution of ranges. (2 marks)

Solution
Shape: Distribution is negatively skewed. Modality: Distribution is unimodal.
Specific behaviours
✓ describes shape
✓ describes modality

See next page

Question 11 **(7 marks)**

The blood pressure of $25\,000$ adults in a large medical study is normally distributed with a mean of 121 mmHg and a standard deviation of 13 mmHg.

(a) The blood pressure of adult A was 153 mmHg and 115 mmHg for adult B. Calculate the standard scores for each adult and hence explain which adult had unusual blood pressure. (3 marks)

Solution
$z_A = \dfrac{153 - 121}{13} \approx 2.46$ $z_B = \dfrac{115 - 121}{13} \approx -0.46$ Adult A had unusual blood pressure - their standard score was more than 2 standard deviations from the mean.
Specific behaviours
✓ shows correct use of standard score formula ✓ both correct standard scores ✓ explains why BP for adult A is unusual

(b) Using the 68%, 95%, 99.7% rule determine the number of adults in the study with blood pressure between 108 and 134 mmHg. (2 marks)

Solution
108 to 134 is $\mu \pm 1$ sd and is middle 68%. $0.68 \times 25000 = 17\,000$ adults.
Specific behaviours
✓ indicates ± 1 sd ✓ correct number

(c) Determine the probability, to three decimal places, that a randomly selected adult from the study had blood pressure of

(i) at least 100 mmHg. (1 mark)

Solution
$p = 0.947$
Specific behaviours
✓ correct probability

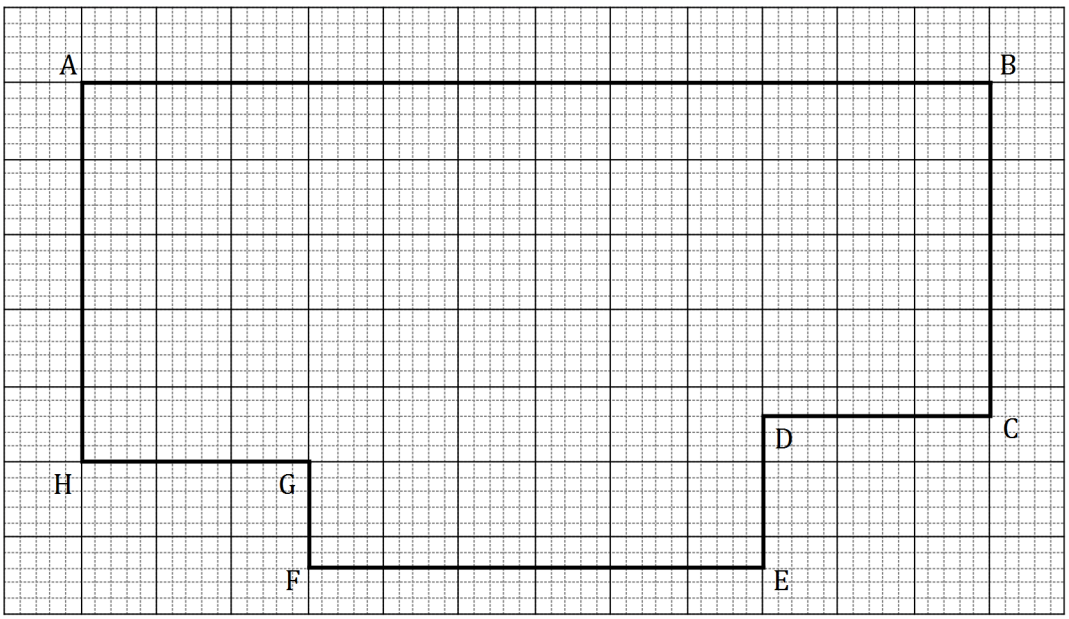
(ii) between 110 and 130 mmHg. (1 mark)

Solution
$p = 0.557$
Specific behaviours
✓ correct probability

See next page

Question 12 **(8 marks)**

The plan shown below on 2 mm grid paper is a $1:50$ scale drawing of a concrete slab.



(a) Show that the length of the slab edge AB is 6 m and calculate the length of the slab edge BC. (2 marks)

Solution
Plan length $AB = 12$ cm and so actual length is $0.12 \times 50 = 6$ m. Plan length $BC = 4.4$ cm and so actual length is $0.044 \times 50 = 2.2$ m.
Specific behaviours
✓ indicates use of plan length and scale for AB ✓ correct length BC

(b) When marking out the slab, a builder measures some of the diagonals to check that the corners are square. Determine the length that diagonal AC should measure. (2 marks)

Solution
$AC^2 = 6^2 + 2.2^2$ $= 40.84$ $AC = 6.39$ m
Specific behaviours
✓ indicates use of Pythagoras' Theorem ✓ correct length, with units

See next page

(c) Determine the volume, in cubic metres, of concrete required for the slab if it must have a uniform thickness of 16 cm. (4 marks)

Solution
Area of bounding rectangle: $A = 6 \times 3.2 = 19.2 \text{ m}^2$ Area HGF: $A = 1.5 \times 0.7 = 1.05 \text{ m}^2$ Area DCE: $A = 1.5 \times 1 = 1.5 \text{ m}^2$ Area of slab: $A = 19.2 - 1.05 - 1.5$ $= 16.65 \text{ m}^2$ Thickness is 16 cm, or 0.16 m. Volume of slab: $V = 16.65 \times 0.16$ $= 2.664 \text{ m}^3$
Specific behaviours
✓ indicates correct use of scale to obtain measurements ✓ indicates appropriate method to obtain slab area ✓ correct area of slab ✓ correct volume in cubic metres

Alternative part solution
Area LH rectangle: $A = 1.5 \times 2.5 = 3.75 \text{ m}^2$ Area centre rectangle: $A = 3 \times 3.2 = 9.6 \text{ m}^2$ Area RH rectangle: $A = 1.5 \times 2.2 = 3.3 \text{ m}^2$ Area of slab: $A = 3.75 + 9.6 + 3.3$ $= 16.65 \text{ m}^2$

See next page

Question 13 **(10 marks)**

The stem plot below shows the percentage scores of students in two classes for the same exam.

Class M		Class N
2	8	2 5
5	7	1 6 7
7 3 3 1	6	0 2 6 6 9
8 7 5 4 3 2 0	5	2 5 7 9
9 8 7	4	1 5 9
9	3	

For the 17 students in Class M, the mean and standard deviation of their scores were 57.2 and 10.3 respectively.

(a) State, with justification, which class had the greater range. (2 marks)

Solution
Class M: $82 - 39 = 43$, Class N: $85 - 41 = 44$ Class N had the greater range.
Specific behaviours
✓ indicates both ranges ✓ correct class

(b) State the number of students in Class N and calculate the mean and standard deviation of the scores of these students. (3 marks)

Solution
$n = 17$ $\bar{x} = 63.1, \quad \sigma_x = 12.3$
Specific behaviours
✓ correct number n ✓ correct mean $\bar{x}$ ✓ correct standard deviation σ

(c) State, with justification, which of the two classes performed better on the exam and explain how this is illustrated by a feature of the stem plot. (3 marks)

Solution
Class N performed better than Class M, as it had a higher mean of 63.1 compared to the mean for Class M of 57.2. The stem plot illustrates that N performed better as the scores for Class N are centred around the $60 - 69$ group whereas the scores for Class M are centred around the $50 - 59$ group.
Specific behaviours
✓ states Class N performed better ✓ justifies by comparing means ✓ explains feature of stem plot

See next page

(d) Compare the spread of scores for the two classes. (2 marks)

Solution
The scores for Class N are more spread out than the scores for Class M, as its standard deviation of 12.3 is higher than the standard deviation of 10.3 for Class M.
Specific behaviours
✓ compares standard deviations ✓ indicates differences in spread

See next page

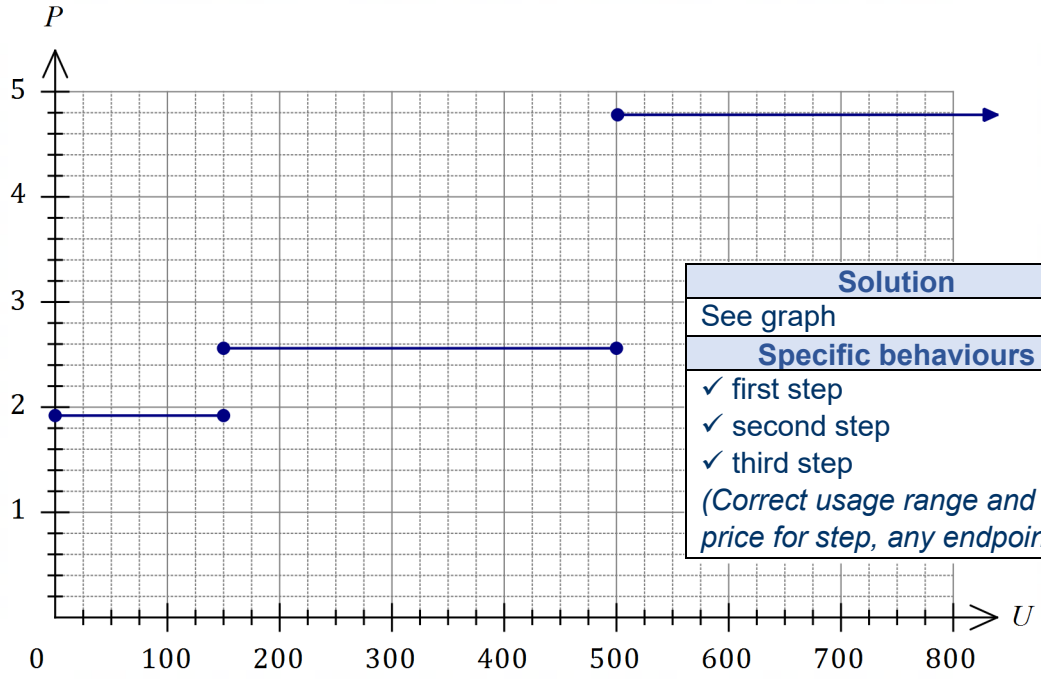
Question 14 **(7 marks)**

The price that a corporation charges its customers for the water they use is shown in the table below.

Usage (U kL)	Up to 150	$150 - 500$	Over 500
Price per kL (P \$)	1.92	2.56	4.78

The usage is based on the customers total usage for the current year. At the end of each year the usage resets to zero.

(a) Draw a graph to show how price P varies with usage U during a year on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ first step ✓ second step ✓ third step *(Correct usage range and price for step, any endpoint)*

(b) A customer's usage at the start of the billing period was 135 kL and at the end it had increased to 208 kL. Calculate the cost of their water usage for this period. (3 marks)

Solution
$150 - 135 = 15, \quad 15 \times 1.92 = 28.80$ $208 - 150 = 58, \quad 58 \times 2.56 = 148.48$ Total: $28.80 + 148.48 = \$177.28$
Specific behaviours
✓ splits into correct usage categories ✓ correct cost for one category ✓ correct total cost

(c) Give a brief reason why the corporation might use these pricing tiers. (1 mark)

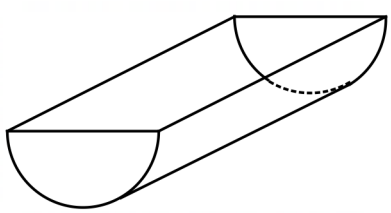
Solution
(Price increases with usage) to encourage customers to conserve water.
Specific behaviours
✓ sensible reason

See next page

Question 15 **(7 marks)**

A water trough can be made from half of a cylindrical oil drum, as shown.

Oil drum P has a radius of 18 cm and a height of 55 cm.



(a) The interior of the trough made from the oil drum P is to be painted with a waterproofing paint. Calculate the area to be painted, rounded to the nearest 10 square centimetres. (3 marks)

Solution
$A_{CYL} = 2\pi(18)(55) + 2\pi(18)^2$ $= 8256.1 \text{ cm}^2$ $A_{\text{Paint}} = 8256.1 \div 2$ $= 4128$ $\approx 4130 \text{ cm}^2$
Specific behaviours
✓ indicates correct use of cylinder SA formula ✓ correct area of cylinder ✓ correct area to be painted, rounded as required

Oil drum Q is a similar figure to oil drum P but with a height of 77 cm.

(b) Obtain a scale factor for the size of the drum Q relative to the drum P. (2 marks)

Solution
$SF = 77 \div 55$ $= 1.4$
Specific behaviours
✓ uses heights ✓ correct scale factor

(c) Determine the capacity of a water trough made from drum Q, given that the capacity of the trough made from the drum P is 28 L. (2 marks)

Solution
$C_Q = 28 \times 1.4^3$ $\approx 76.8 \text{ L}$
Specific behaviours
✓ uses cube of scale factor ✓ correct capacity

See next page

Question 16 **(8 marks)**

(a) A borrower secures a short term loan for an amount of $\$2\,350$ for 28 days at a simple interest rate of 12.97% pa. Determine the amount that must be repaid. (3 marks)

Solution

$$I = 2350 \times \frac{12.97}{100} \times \frac{28}{365}$$
$$= \$23.38$$

$$\text{Repayment} = 2350 + 23.38$$
$$= \$2373.38$$

Specific behaviours

✓ uses simple interest formula, adjusting for days
✓ calculates interest
✓ correct repayment, to nearest cent

(b) Another borrower is considering a loan of $\$18\,500$ for 3 years to use as part payment for a car. The car dealership offer the borrower a loan at 7.94% pa compounded monthly. The borrowers bank offer a loan at 8.19% pa compounded annually.

Determine which loan will be cheaper for borrower to repay at the end of the 3 year period and state how much they save compared to the other loan. (5 marks)

Solution

Car dealer:

$$A = 18500\left(1 + \frac{7.94}{12 \times 100}\right)^{12\times 3}$$
$$= \$23\,457.40$$

Bank:

$$A = 18500\left(1 + \frac{8.19}{100}\right)^{3}$$
$$= \$23\,427.89$$

Difference:

$$23457.40 - 23427.89 = \$29.51$$

The bank loan will be cheaper by $\$29.51$.

Specific behaviours

✓ uses compound interest formula for dealer, adjusted for monthly
✓ correct amount to repay to dealer
✓ uses compound interest formula for bank
✓ correct amount to repay to bank
✓ states cheaper loan and saving

See next page

Question 17 **(7 marks)**

Police in a city have detected that a mobile phone lies within a triangle bounded by three phone towers P, Q and R where tower R lies due east of tower P and tower Q is the most northerly tower. The distances PQ, QR and PR are $650, 540$ and 810 metres respectively.

(a) Use Heron's rule to determine the area of the triangle within which the phone has been located. (3 marks)

Solution

$$s = \frac{650 + 540 + 810}{2} = 1000$$

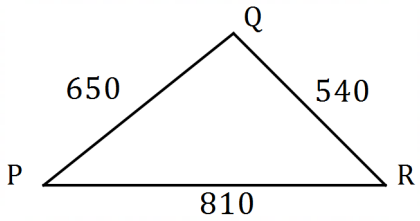
$$A = \sqrt{1000^2 + (1000 - 650)^2 + (1000 - 540)^2 + (1000 - 810)^2}$$
$$= \sqrt{1000^2 + 350^2 + 460^2 + 190^2}$$
$$\approx 174900 \text{ m}^2$$

Specific behaviours

✓ semi-perimeter
✓ substitutes correctly
✓ correct area

(b) Determine the bearing of tower Q from tower P to the nearest degree. (4 marks)

Solution



$$\cos P = \frac{650^2 + 810^2 - 540^2}{2(650)(810)}$$
$$P \approx 42°$$

$$\text{Bearing} = 90° - 42° = 048°$$

Specific behaviours

✓ diagram, roughly to scale, R east of P
✓ correct use of cosine rule
✓ angle P
✓ correct bearing

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Question 18 **(7 marks)**

A store can buy cartons containing packs of $10, 25$ and 50 tea bags at the wholesale prices shown below.

Pack size	10	25	50
Packs per carton	200	80	40
Wholesale cost price per carton ($)	256.00	236.00	212.00
Retail selling price per pack ($)	3.55	5.75	10.95

(a) Calculate the percentage profit that the store makes selling a pack of 25 tea bags. (2 marks)

Solution
Cost per pack: $C = 236 \div 80 = 2.95$ % profit per pack: $P = \frac{5.75 - 2.95}{2.95} \times 100 = 94.9\%$
Specific behaviours
✓ cost per pack ✓ percentage profit

(b) All prices shown in the table include GST at 10%. Determine the cost price of a pack of 50 tea bags without GST. (2 marks)

Solution
Cost per pack: $C_{INC} = 212 \div 40 = 5.30$ Cost ex GST: $C_{EX} = 5.30 \div 1.10$ $= \$4.82$
Specific behaviours
✓ cost per pack ✓ price ex GST

(c) Use the unit cost method to rank the pack sizes from best to worst value based on their retail prices when the packs of 10 tea bags are sold with a 40% discount. (3 marks)

Solution
10 pack: $3.55 \times 0.60 \div 10 = 21.3$c 25 pack: $5.75 \div 25 = 23$c 50 pack: $10.95 \div 50 = 21.9$c Best to worst value: 10 pack, 50 pack, 25 pack.
Specific behaviours
✓ 10 pack price per tea bag ✓ other unit prices ✓ ranks correctly

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Question 19 **(7 marks)**

A person who has a part time job qualifies for a government allowance of $\$675$ per fortnight, but this allowance is reduced by 50 cents in the dollar for fortnightly earnings over $\$240$.

The person, who earns $\$21.50$ per hour, has created the spreadsheet below.

	A	B	C	D	E	F
1	0	0.00	0.00	675.00	675.00	337.50
2	5	215.00	0.00	675.00	890.00	445.00
3	10	430.00	95.00	580.00	1010.00	505.00
4	15	645.00	202.50	472.50	1117.50	558.75
5	20	**860.00**	**310.00**	**365.00**	**1225.00**	**612.50**

Key to columns:
A: Weekly hours worked
B: Fortnightly earnings
C: Reduction in government allowance
D: Allowance received
E: Total fortnightly income (earnings plus allowance)
F: Total weekly income

(a) Using the space below to show all your working, complete row 5 of the spreadsheet. (5 marks)

Solution
B5: $20 \times 21.50 \times 2 = 860.00$ C5: $(860.00 - 240.00) \times 0.5 = 310.00$ D5: $675.00 - 310.00 = 365.00$ E5: $860.00 + 365.00 = 1225.00$ F5: $1225 \div 2 = 612.50$
Specific behaviours
✓✓✓✓✓ each correct value

(b) By referring to the number in a cell in the spreadsheet using its column letter followed by its row number (e.g. E2 refers to the cell containing the number 890.00) write a suitable formula to

(i) calculate the value in B5 from the value in A5. (1 mark)

Solution
$B5 = A5 * 21.50 * 2$
Specific behaviours
✓ any correct formula referring to $A5$

(ii) calculate the value in F5 from the value in B5. (1 mark)

Solution
$F5 = (675 - (B5 - 240) * 0.5 + B5)/2$
Specific behaviours
✓ any correct formula referring to $B5$

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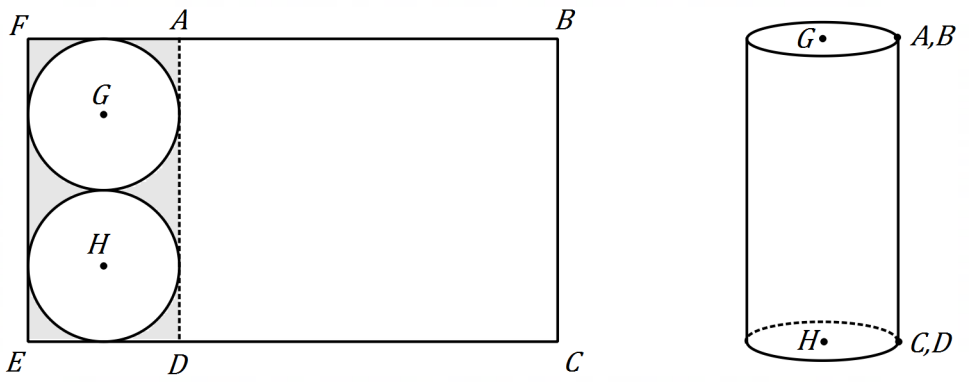
Question 20 **(9 marks)**

A company fabricates cylindrical storage tanks.

A thin rectangular sheet of steel $FBCE$ with $FB = 4.97$ m and $FE = 2.40$ m is cut into two rectangles along AD, where $AF = \frac{1}{2}FE$.

Two circles of radius $\frac{1}{2}AF$ and with centres G and H are cut out of the smaller rectangle for the ends of the cylinder.

The remaining rectangle $ABCD$ is then rolled up and edge AD is welded to edge BC to form the wall of the cylinder. Finally, the circular ends are welded in place.



(a) Determine the area of the sheet $FBCE$ that is not used in the fabrication of a tank.

(3 marks)

Solution
$A_{AFED} = 1.2 \times 2.4$ $= 2.88 \text{ m}^2$ $A_{Circle} = \pi(0.6)^2$ $= 1.131 \text{ m}^2$ $A_{Not\ Used} = 2.88 - 2(1.131)$ $= 0.618 \text{ m}^2$
Specific behaviours
✓ area of rectangle ✓ area of circle ✓ area not used

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(b) Calculate the capacity of the finished cylinder in litres, given that 1000 L is equal to one cubic metre. (2 marks)

Solution
$V = \pi(0.60)^2(2.40)$ $= 2.714 \text{ m}^3$ Capacity: $2.714 \times 1000 = 2714$ L.
Specific behaviours
✓ correct volume ✓ correct capacity

(c) Fabrication costs are $\$45.75$ for the sheet $FBCE$, $\$4.50$ per metre for all cuts that must be made and $\$9.50$ per metre for all welds needed to join edges together. Determine the cost of fabricating one tank. (4 marks)

Solution
Length of cuts: $C_{Circle} = 2\pi(0.60)$ $= 3.77 \text{ m}$ $\text{Cuts} = AD + 2 \times \text{circles}$ $= 2.40 + 2(3.77)$ $= 9.94 \text{ m}$ Length of welds equals length of cuts. $\text{Cost} = 45.75 + 4.50 \times 9.94 + 9.50 \times 9.94$ $= 45.75 + 44.73 + 94.43$ $= \$184.91$
Specific behaviours
✓ circumference of one circle ✓ length of cuts ✓ indicates correct method to obtain cost ✓ correct cost

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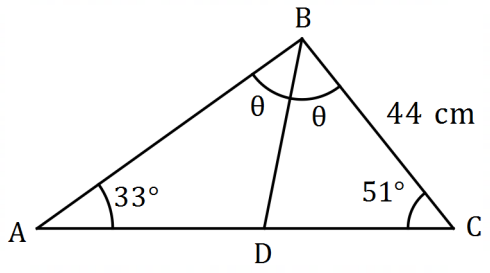
Question 21 **(8 marks)**

A triangular logo ABC is to be mounted at the entrance to a company. ΔABC is split into two smaller triangles by line BD that bisects $\angle ABC$, as shown in the sketch. ΔABD is coloured red and ΔBDC is grey.

The length of BC is 44 cm, $\angle BAC = 33°$ and $\angle BCA = 51°$.



(a) Determine the length of AB. (2 marks)

Solution
$\frac{AB}{\sin 51°} = \frac{44}{\sin 33°}$ $AB = 62.8 \text{ cm}$
Specific behaviours
✓ correct use of sin rule ✓ correct length

(b) Determine the length of BD. (3 marks)

Solution
$2\theta = 180° - 33° - 51° = 96° \Rightarrow \theta = 48°$ $\angle BDC = 180° - 51° - 48° = 81°$ $\frac{BD}{\sin 51°} = \frac{44}{\sin 81°}$ $BD = 34.6 \text{ cm}$
Specific behaviours
✓ calculates θ ✓ correct use of sin rule ✓ correct length

(c) Determine what percentage of the logo is coloured red. (3 marks)

Solution
$A_{ABC} = 0.5(62.8)(44)\sin 96°$ $= 1374 \text{ cm}^2$ $A_{ABD} = 0.5(62.8)(34.6)\sin 48°$ $= 808 \text{ cm}^2$ $\%\text{ red} = \frac{808}{1374} \times 100 \approx 59\%$
Specific behaviours
✓ area of whole ✓ area of smaller triangle ✓ correct percentage

End of questions